Steady Incompressible Variable-Thickness Shear Layer Aerodynamics

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Theme

THE parallel constant-thickness shear flow model used in problems of hydrodynamic stability, panel flutter, and water wave generation by wind was employed recently by Ventres[1] to account approximately for the presence of a boundary layer over a thin lifting flat plate. The shear flow model has proved to be very valuable in nonlifting problems.[2] For lifting problems, Melnik and Chow[3] have shown, by considering the triple-deck structure iniscid-viscous interaction problem, that only the shear layer model rather than the fully viscous boundary layer is required for predicting the pressure distribution (at least) near the trailing edge of a flat plate. This synoptic extends Ventres' constant-thickness shear layer theory to a shear layer of slowly varying thickness which presumably is a better approximation to a real boundary layer.

Contents

Detailed order-of-magnitude analysis given in Ref. 4 shows that the perturbation pressure $p(z,y,z)$ due to airfoil motion in a three-dimensional steady incompressible shear flow (slowly varying in streamwise coordinate x) satisfies

$$\nabla^2 p - (2/U)(\partial U/\partial z)(\partial p/\partial z) = 0 \tag{1}$$

where the mean flow velocity U is assumed to be

$$U = U_I \qquad z \geq \delta(x,y)$$
$$U = U_I[z/\delta(x,y)]^{1/N} \qquad z \leq \delta(x,y)$$

U_I is freestream velocity, $\delta(x,y)$ is boundary-layer thickness, and z,y are transverse and spanwise coordinates, respectively.

The associated boundary conditions are, on the solid surface,

$$\lim_{z_0 \to 0} \left. \frac{\partial p}{\partial z} \right|_{z=z_0} = -\rho U(z=z_0)\frac{\partial w}{\partial x} \tag{2}$$

and, at infinity (the finiteness condition),

$$p \to 0 \text{ as } z \to \infty \tag{3}$$

where ρ is the fluid density and

$$w_{|z=z_0} = U_{|z=z_0} \cdot (\partial f/\partial x)$$

where f is the airfoil shape. While allowing a discontinuity in the mean flow velocity gradient across $z=\delta(x,y)$, we impose the continuity of the pressure and the pressure gradient at $z=\delta(x,y)$:

$$p[x,y,z=\delta^-(x,y)] = p[x,y,z=\delta^+(x,y)] \tag{4a}$$

$$(\partial p/\partial z)[x,y,z=\delta^-(x,y)] = (\partial p/\partial z)[x,y,z=\delta^+(x,y)] \tag{4b}$$

In view of the slowly varying behavior, it is convenient to write the boundary-layer thickness as $\delta=\delta(\epsilon\xi,\epsilon'\eta)$, where $\epsilon << 1$ and $\epsilon' << 1$. Here, ϵ and ϵ' are nondimensional parameters characterizing the slow variation of the boundary-layer thickness in the x and y directions, respectively. ξ and η are nondimensional coordinates defined as $\xi \equiv x/\delta_{max}$, $\eta = y/\delta_{max}$, $\zeta = z/\delta(\epsilon\xi,\epsilon'\eta)$, where δ_{max} is the trailing edge boundary-layer thickness. The definition for ζ has the advantage that all boundary conditions are applied on constant values of ζ, e.g., $\zeta=0$, $=1$, and $\to\infty$ [corresponding to $z=0$, $\delta(x,y)$, and $\to\infty$].

Consider a wavy wall whose profile is described by the real part of the complex function $f(x,y) = \bar{f}\exp[i(\alpha x+\gamma y)] = \bar{f}\exp[i(\bar{\alpha}\xi+\bar{\gamma}\eta)]$ where $\bar{\alpha}=\alpha\xi_{max}$, $\bar{\gamma}=\gamma\delta_{max}$. The wavy wall will generate a perturbation pressure field of the following form (see Ref. 5 for the method of multiple scales):

$$p = \bar{p}(\zeta,\epsilon\xi,\epsilon'\eta;\bar{\alpha},\bar{\gamma};\epsilon,\epsilon')\exp[i(\bar{\alpha}\xi+\bar{\gamma}\eta)]$$

We further assume the following series expansion for $\bar{p}$, since $\epsilon << 1$ and $\epsilon' << 1$:

$$\bar{p}(\zeta,\epsilon\xi,\epsilon'\eta;\bar{\alpha},\bar{\gamma};\epsilon,\epsilon') = \bar{p}_0(\zeta,\epsilon\xi,\epsilon'\eta;\bar{\alpha},\bar{\gamma})$$
$$+\epsilon\bar{p}_I(\zeta,\epsilon\xi,\epsilon'\eta;\bar{\alpha},\bar{\gamma}) + \epsilon'\bar{p}_I(\zeta,\epsilon\xi,\epsilon'\eta;\bar{\alpha},\bar{\gamma}) + \text{H.O.T}$$

One can show[4] that the lowest-order governing equation for $\bar{p}$ is

$$\frac{\partial \bar{p}_0}{\partial \zeta^2} - \frac{2}{N\zeta}\frac{\partial \bar{p}_0}{\partial \zeta} - \left[\left(\frac{\delta}{\delta_{max}}\right)^2 \bar{R}^2\right]\bar{p}_0 = 0 \tag{5}$$

where δ is the variable boundary-layer thickness, and the associated boundary conditions are

$$\left.\frac{\partial \bar{p}_0}{\partial \zeta}\right| z=z_0 = \rho\left(\bar{\alpha}\frac{\delta}{\delta_{max}}\right)^2 U^2\left(\frac{z_0}{\delta}=\zeta_0\right)\left(\frac{\delta_{max}}{\delta}\frac{\bar{f}}{\delta_{max}}\right) \text{ as } \zeta_0 \to 0 \tag{6}$$

$$\frac{\partial \bar{p}_0}{\partial \zeta} + \left(\frac{\delta}{\delta_{max}} \cdot \bar{R}\right)\bar{p}_0 = 0 \quad \text{at } \zeta = 1 \tag{7}$$

From Eqs. (5-7), the lowest-order wall solution is

$$(\tfrac{1}{2}\pi)^2(W^*/U_I) = \bar{K}(\bar{p}_w/\rho U_I^2)[\alpha,\gamma;\delta(x,y)] \tag{8}$$

where $\bar{K}$ is the same as Ventres' kernel,[1] with his uniform thickness replaced by the variable $\delta(x,y)$. W^* is defined as $i\alpha U_I\bar{f}$. Because of the dependence of $\bar{K}$ and $\bar{p}_w$ on $\delta(x,y)$, the convolution theorem does not apply directly to Eq. (8). However, when $\delta(x,y)$ is not anywhere near zero, it can be

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Index category: Aircraft Aerodynamics (including Component Aerodynamics).

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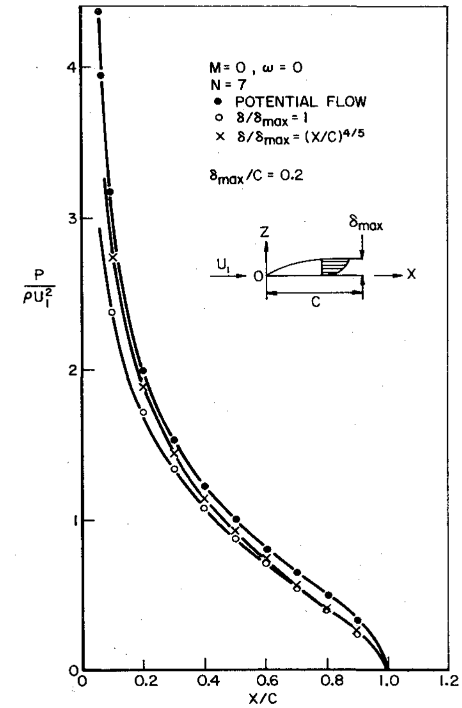

Fig. 1 Pressure distribution along chord.

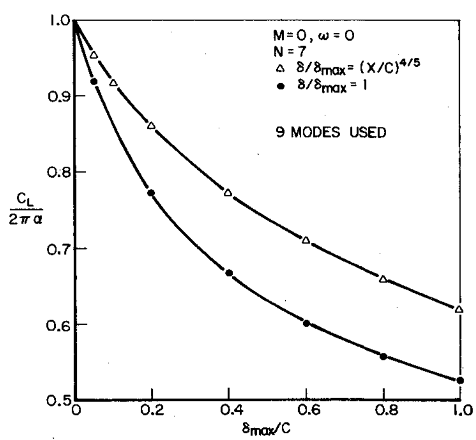

Fig. 2 Lift coefficient vs δ_{max}/c for both uniform and variable boundary-layer thickness of the case $N=7$.

shown[4] that the wall pressure P_w can be calculated by

$$P_w = P_{w0} + \{ 1 - [\delta(x,y)/\delta_{max}] \} P_{w1} + 0\{ 1 - [\delta(x,y)/\delta_{max}] \}^2 \tag{9}$$

where P_{w0} and P_{w1} are solutions of the following integral equations:

$$\frac{W}{U_1}(x,y) = \iint_{wing\ surface} K_A(x-x', y-y') \frac{p_{w0}}{\rho U_1^2}(x',y')\,dx'\,dy' \tag{10}$$

$$\frac{W_{eq}}{U_1}(x,y) = \iint_{\text{wing surface}} K_A(x-x', y-y') \frac{p_{w1}}{\rho U_1}(x',y')\,dx'\,dy' \tag{11}$$

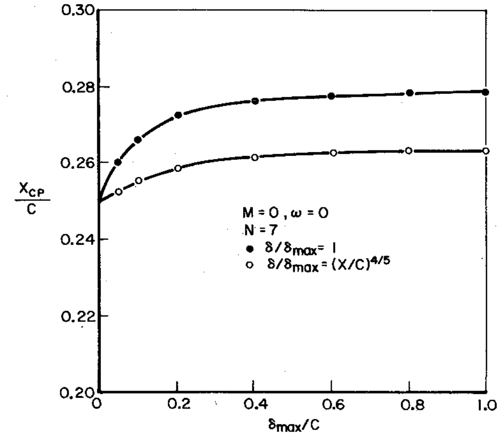

Fig.3 Position of center of pressure vs δ_{max}/c for the case $N=7$.

where W_{eq} is an equivalent upwash in terms of the lowest-order solution $P_{w0}(x,y)$, i.e.,

$$\frac{W_{eq}}{U_1}(x,y) = \iint_{\text{wing area}} -K_B(x-x', y-y') \frac{p_{w0}}{\rho U_1^2}(x',y')\,dx'\,dy' \tag{12}$$

The domains of integration of the integrals in Eqs. (10-12) cover only the wing surface area because the pressure is zero everywhere off the wing surface for the lifting surface problem. These integral equations have been solved by the same numerical techniques employed in potential flow lifting surface theory.[4]

In Figs. 1 and 2, results are shown for typical chordwise pressure distributions and total lift coefficients for a two-dimensional flat plate airfoil in potential flow and also constant δ and variable δ shear layers. In general, the aerodynamic load is decreased by the shear layer as compared to the potential flow, whereas the variable thickness shear layer decreases it less than the uniform thickness shear layer based upon equal maximum thicknesses. Furthermore, Fig. 1 indicates less pressure reduction near the leading edge for the variable δ case compared to the larger reduction for the constant δ case. As a result, the variable shear layer moves the center of pressure backwards by a smaller amount than the constant shear layer, as shown in Fig. 3.

The present theory for the variable-thickness problem has been worked out in detail for the steady, incompressible case. Its extension to three-dimensional, unsteady compressible flows seems to be workable.

References

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